SIMPLE AND SHOCK WAVES IN NONLINEAR HIGH-INTENSITY NONSTATIONARY PROCESSES OF HEAT AND MASS TRANSFER

P. M. Kolesnikov

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Solutions of the equations of nonlinear high-intensity nonstationary heat and mass transfer of the simple wave type are investigated. The appearance of heat- and mass-transfer shock waves is predicted. The conditions of conversion of simple into shock waves are discussed. A method is proposed for determining the thermophysical properties of materials from the structure of the simple waves.

We consider the hyperbolic equation of nonstationary high-intensity heat and mass transfer proposed by Luikov [1]:

$$\frac{\partial^2 T}{\partial \tau^2} = w_r^2 \frac{\partial^2 T}{\partial x^2}, \tag{1}$$

where T is the temperature in heat-transfer processes or the concentration in mass transfer, x is a coordinate, τ is the time, and w_r is the rate of propagation of heat or mass. To be specific, in what follows we consider heat-propagation processes. The argument is equally applicable to mass transfer processes.

The rate of heat propagation is given by the expression

$$w_r = \sqrt{\frac{\lambda}{c\gamma\tau_r}} \tag{2}$$

and depends on the thermophysical properties: the specific heat c, the density of the medium γ, the thermal conductivity λ, and the heat-propagation time constant τ_r or relaxation time. If w_r is constant, then Eq. (1) is a linear hyperbolic equation with constant coefficients and describes the propagation of thermal waves in the medium at a finite velocity w_r. The general solution of Eq. (1) represents two traveling waves

$$T = f_1(x - w_r\tau) + f_2(x + w_r\tau), \tag{3}$$

propagating without distortion of the profile at constant velocities w_r in opposite directions. The specific form of the arbitrary functions f_1 and f_2 defining the wave shape is found from the boundary conditions by the methods of mathematical physics.

Since the thermophysical properties λ, c, γ, τ_r may be functions of temperature, the coordinates, time, etc., in the general case w_r may depend in a complex fashion on many variables.

$$w_r = w_r\left(T, \frac{\partial T}{\partial x}, \frac{\partial T}{\partial \tau}, x, \tau, \ldots\right). \tag{4}$$

Therefore, in the general case, Eq. (1) is a nonlinear equation of heat and mass transfer for intense nonstationary processes,

We consider Eq. (1) without making any assumptions with respect to Eq. (4). Whatever the form of Eq. (4), Eq. (1) can be reduced to a system of ordinary differential equations by the methods of the theory of characteristics [2, 3]. Equation (1) is completely equivalent to a system of two families of characteristics:

1) the characteristics of the first family

$$dx = +w_r d\tau \tag{5}$$

with the conditions

$$dT_\tau = w_r dT_x; \tag{6}$$

2) the characteristics of the second family

$$dx = -w_r d\tau \tag{7}$$

with the conditions

$$dT_\tau = -w_r dT_x. \tag{8}$$

By covering the field with a sufficiently dense network of characteristics of the first and second families, from known boundary conditions we can find the solution of (1) at any point of the field, if such a solution exists. The equations of the characteristics [(5)–(8)] can be used for the effective calculation of the thermal waves for any relation (4).

Examining system (5)–(8), we note that in the nonlinear case if the propagation velocity of the thermal wave depends only on $T_X = dT/dx$, i. e.,

$$w_r = w_r(T_x), \tag{9}$$

condition (6) does not depend on Eq. (5) and can be integrated separately, which gives the first integral

$$T_\tau - \int w_r(T_x)\, dT_x = \xi, \tag{10}$$

where ξ is a constant of integration taking different values for each characteristic (5).

Similarly, condition (8) does not depend on characteristic (7), and its integration gives another first integral

$$T_\tau + \int w_r(T_x)\, dT_x = \eta, \tag{11}$$

where η is a constant of integration different for each characteristic of the second family (7).

Thus, in the given case the problem has been reduced to the integration of two ordinary differential equations (5) and (7) with conditions (10) and (11). Integrating (5), we find

$$x = w_r(T_x)\,\tau + F_1(T), \tag{12}$$

where F_1 is an arbitrary function of T. Integration of (7) gives

$$x = -w_r(T_x)\,\tau + F_2(T), \tag{13}$$

where F_2 is an arbitrary function of T.

Equation (12) with condition (10) and Eq. (13) with condition (11) form two families of solutions, which by analogy with gasdynamics [4, 5] may be called traveling waves of finite amplitude or simple waves of heat and mass transfer.

Under certain conditions the simple waves of finite amplitude of the theory of heat and mass transfer, like waves of finite amplitude in a gas [4, 5] or in nonlinear electrodynamics [6, 7], may be transformed into shock waves of heat and mass transfer. We consider the conditions under which simple waves are converted into shock waves, assuming relation (9). To be specific, we select a wave propagating in one direction, for example, in the positive direction. The propagation velocity of a simple wave at any point is determined by the temperature gradient

$$\frac{dx}{d\tau} = w_r(T_x). \tag{14}$$

If the temperature gradient is such that when dT/dx changes in the thermal wave the propagation velocity w_r decreases, subsequent simple waves do not overtake the preceding ones, and simple waves are propagated in the nonlinear medium. If, however, the change in temperature gradient is such that subsequent traveling simple waves have propagation velocities greater than those of the preceding waves, they overtake the latter, as a result of which shock heat and mass transfer waves are formed.

As distinct from (14), it is possible to consider a more general case of relation (4), for which Eqs. (5)–(8) are also valid. Thus, assuming that

$$w_r = w_r(T), \tag{15}$$

from Eq. (5) for a given temperature profile it is also possible to arrive at the concept of shock waves, if a change of temperature T leads to an increase in the propagation velocity of the thermal wave.

To study the structure of the thermal shock wave it is first necessary to specify the form of relation (7). In general form, this relation can obviously be represented as an integrodifferential equation, and by investigating its solution, using (5)–(8), we can determine the structure of the thermal shock wave. Thus, the most important problem is to establish the specific relationships of the material characteristics (4).

If we instantaneously apply to the end of a rod made of heat-conducting material with nonlinear properties a temperature or heat flux of finite magnitude, then a simple centered wave travels along the rod (it is assumed that shock waves are not formed). If in a certain section we register the quantities $T_\tau = T_\tau(\tau)$ and $T_x = T_x(\tau)$ as functions of time, then from Eqs. (5) and (6) for the centered wave we can write

$$w_r = \frac{x}{\tau}, \tag{16}$$

$$w_r = \frac{dT_\tau(\tau)}{dT_x(\tau)} = f(x, \tau), \tag{17}$$

where $f(x, \tau)$ is a certain function of time. Equations (16), (17) may be regarded as a parametric representation of the propagation velocity of the thermal wave.

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Khar'kov Aviation Institute